Heat Engines (p118)

Q1: A steam-cycle power station has a boiler temperature of 520°C and a condenser temperature of 21°C. What is the maximum (theoretical) efficiency? (%)

Max theoretical efficiency = ideal = Carnot

Carnot efficiency: (Ideal efficiency)

\[ \eta_c = 1 - \frac{T_c}{T_h} \]

where

- \( T_h \) = the hot temperature (Kelvin) that the heat flows FROM
- \( T_c \) = the cold temperature (Kelvin) that the heat flows TO

\[ T_h = 520 + 273 = 793 \text{ K} \]
\[ T_c = 21 + 273 = 294 \text{ K} \]
\[ n = 1 - \frac{294}{793} = 0.6293 (63\%) \]

More likely \( T_2 = 150 \)
\[ T_c = 273 + 150 = 423 \]
\[ n = 1 - \frac{423}{795} = 0.4679 (47\%) \]

Q3: Solar powered Stirling Engine: A mirror of diam 12 m focusses sunlight (599 W/m²) to achieve maximum temperature of 717°C. Air is at 21°C.

(a) Find theoretical efficiency (%)

\[ \eta_c = 1 - \frac{T_c}{T_h} \]

Theoretical eff = \( 1 - \frac{(273+21)}{(273+717)} \)
\[ = 0.703 \ (70.3\%) \]

(b) Find heat supply rate.

Heat from sun = Area of mirror * heat from sun
\[ = \pi * 6^2 * 599 \]
\[ = 113.097 * 599 = 67745 \text{ W} \]

(c) Find theoretical power output.
Work Output = Heat Supplied - Heat Rejected
\[ W = Q_s - Q_R \]

So efficiency
\[ \eta = \frac{W}{Q_s}, \text{ So } W = \eta Q \]
\[ = 0.703 \times 67745 = 47624.735 \text{ W} \]
Q6: An internal combustion engine uses fuel with energy content of 37.1 MJ/kg at a rate of 4.2 kg/hour. Efficiency is 30%.

(a) What is the power output?
Heat supplied: \( Q_s = mE \)
m = mass/sec
\[ = \frac{4.2}{3.600} = 1.1667 \times 10^{-3} \text{ kg/s} \]
\[ Q_s = 1.1667 \times 10^{-3} \times 37.1 \times 10^6 = 43284.57 \text{ J/s (W)} \]
\[ W = \eta Q \]
\[ = 0.30 \times 43284.57 = 12985.4 \text{ Joules/s (W)} \]

(b) What is the rate of heat generated?
\[ Q_S = W + Q_R \]
\[ Q_R = Q_S - W = 43284.57 - 12985.4 = 30299.17 \text{ J/s (W)} \]
Otto Cycle P-V

Q8: Otto cycle; Compression ratio = 8.3:1, inlet = 96.8kPa (abs) at 54°C. Heat supply = 945 kJ/kg of air.
(a) Find compression temperature T2 (K).

Note: This question is worked out for each kg of air.
(specific power)
Identify the gas process.
Adiabatic compression

Convert:

\[ T_1 = 54 + 273.15 = 327.15 \, \text{K} \]

\( \frac{V_1}{V_2} \) = compression ratio

\[ = 8.3 \]

\[ T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{(n-1)} \]

\[ T_2 = 327.15 \times 8.3^{(1.4-1)} \]

\[ = 762.743 \, \text{K} \, (490 \, ^\circ \text{C}) \]

And... for fun...

\[ P_2 = P_1 \left( \frac{T_2}{T_1} \right)^{\frac{n}{n-1}} \]

\[ = 1.8732 \times 10^6 \, \text{Pa} \]

(b) Find maximum temperature T3 (K).
Identify the gas process. Constant volume heating (Isochoric)

\[ Q = mc_v(T_3 - T_2) \]

\[ T_3 = T_2 + \frac{Q}{mc_v} \]

\[ = 762.743 + 945000 / (1 \times 718) \]

\[ = 2078.9 \, \text{K} \]

\[ P_3 = P_2 \times T_3 / T_2 \]

\[ = 1.8732 \times 10^6 \times 2078.9 / 762.743 \]

\[ = 5.1055 \times 10^6 \, \text{Pa} \]
Q10: (cont) Otto cycle; Compression ratio = 8.3:1, inlet = 96.8kPa (abs) at 54°C. Heat supply = 945 kJ/kg of air.
(c) Find expanded temperature T4 (K).

Adiabatic expansion...

T4 = T3*(V3/V4)^(n-1)

Expansion ratio

1/8.3 = 0.1205

T3 = 2078.899 K

So...

T4 = T3*(V3/V4)^(n-1)

= 2078.899*0.1205^0.4

= 891.7194 K

and... (for fun)

P4 = P3*(T4/T3)^(n/(n-1))

= 5.1055E6 * (891.7194/2078.9)^(1.4/0.4)

= 263888 Pa

Q11: (cont) Otto cycle; Compression ratio = 8.3:1, inlet = 96.8kPa (abs) at 54°C. Heat supply = 945 kJ/kg of air.
(d) Find indicated work (per kg of air).

Indicated power (no friction losses)

Power = Work/sec

W_{\text{Indicated}} = W_{\text{Expand}} - W_{\text{Compress}}

W_{\text{Expand}} = p3V3-p4V4/(n-1)

But we don't know the VOLUME!
(See example p137-138)

Use heat rejected...

W = Q_s-Q_R

Q_s = heat gain from 2 to 3 (const volume)
Q = 945000 J/kg (from fuel)
Q_R = heat loss from 4 to 1
Q = 945000 J/kg (from fuel)
Q_R = heat loss from 4 to 1
Q_R = mc_v(T1-T4)
   = 1 *718*(327.15 - 891.7194)
   = -405361 J/kg
W = Q_S - Q_R
   = 945000-405361
   = 539639 J/kg
(e) Find theoretical efficiency (%).

\[ \eta = 1 - \frac{Q_R}{Q_S} = 1 - \frac{Q_R}{Q_S} \]
\[ \eta = 1 - \frac{405361}{945000} = 0.571 \]
or...
\[ \eta = \frac{W}{Q_S} = \frac{539639}{945000} = 0.571 \]
Q16: A steam engine outputs $54\ kW$ using wood (energy content $19.8\ \text{MJ/kg}$) at a rate of $25\ \text{kg/hour}$. Steam is at $520^\circ\text{C}$ and cooling water is at $20^\circ\text{C}$. (a) What is the overall efficiency? ($\%$)

$$\eta = \frac{W}{Q_s}$$

$Q_s = mE$

Q17: (cont) A steam engine outputs $54\ kW$ using wood (energy content $19.8\ \text{MJ/kg}$) at a rate of $25\ \text{kg/hour}$. Steam is at $520^\circ\text{C}$ and cooling water is at $20^\circ\text{C}$. (b) Find the maximum (theoretical) efficiency. ($\%$)

Max theoretical efficiency = ideal = Carnot

Carnot efficiency: (Ideal efficiency)

$$\eta_C = 1 - \frac{T_c}{T_h}$$

where

$T_h = \text{the hot temperature (Kelvin) that the heat flows FROM}$

$T_c = \text{the cold temperature (Kelvin) that the heat flows TO}$
Find the heat supply rate...

\[ W, \eta, \dot{Q}_s \]

\[ \dot{Q}_s = \frac{W}{\eta} \]

\[ = 1.08 \times 10^6 / 0.011792 \]
\[ = 91646786 \text{ W} \quad (91.6 \text{ MW... Wow!}) \]

Find hot seawater flow rate. (kg/s)

The hot seawater heats the ammonia up to within 3° of the hot incoming seawater temperature. That means the seawater itself is cooled down by 3 degrees by the time it comes out of that heat exchanger. (This assumes both fluids exit the heat exchanger at the same temperature - which is
approximately correct here - not that we are told anything else anyway, so we have to assume it in this case.)

From $\dot{Q} = \dot{m}c\Delta T$
then $\dot{m} = \dot{Q}/(c\Delta T)$

$\dot{m} = 91646786/(4\times1000\times3)$
$= 7637.23 \text{ kg/s} \ (7.6 \text{ tonnes per sec!})$

Find heat rejection rate.

$W = Q_s - Q_R$

Simply take away the work from the supplied heat. (We can also do this in rate - per second - format too)

$QR = QS - W$

$QR = 91646786 - 1080000$

$= 90566786 \text{ W} \ (90.5 \text{ MW})$

Find cold seawater flow rate. (kg/s).

Same as the hot water (heat source), the cold seawater (heat rejector) cools the ammonia down to within $3^\circ$ of the cold incoming seawater temperature. That means the seawater itself is heated up by 3 degrees by the time it comes out of that heat exchanger. (Assuming both fluids exit the heat exchanger at the same temperature again)

From $\dot{Q} = \dot{m}c\Delta T$
then $\dot{m} = \dot{Q}/(c\Delta T)$

$\dot{m} = 90566786/(4\times1000\times3)$

$= 7547.23 \text{ kg/s} \ (7.5 \text{ tonnes per sec!})$
Heat Fluid = T1 + 3°C

COLD WATER
IN = T1