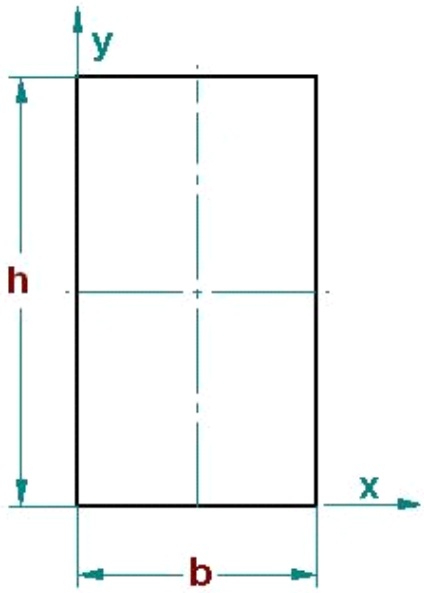
Second Moment of Area

Tuesday, 26 July 2011
8:35 PM

Second Moment of Area.
Ixx = Second moment of Area about the x-x axis



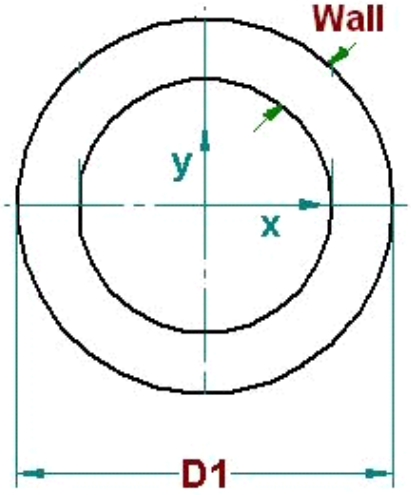
Q2: Find Ixx, where b = 6.9 mm and h = 13 mm.

Ixx = $bh^3/12$ = 6.9*13^3/12 = 1263.275 mm^4

Q3: (cont) Find Iyy, where b = 6.9 mm and h = 13 mm.

Iyy = $hb^3/12$ = 13*6.9^3/12 = 355.8848 mm4

Q7: Find Ixx, D1 = 13 mm and Wall thickness = 1.7 mm



Do outside (solid shaft)
Ixx = $\pi*D^4/64$ = Pi*13^4/64 = 1401.985 mm4
Do inside (the hole)
ID = 13-2*1.7 = 9.6
Ixx = $\pi*D^4/64$ = Pi*9.6^4/64 = 416.922 mm4
Net Ixx = 1401.985 - 416.922 = 985.063 mm4

Parallel Axis Theroem

Tuesday, 13 March 2012
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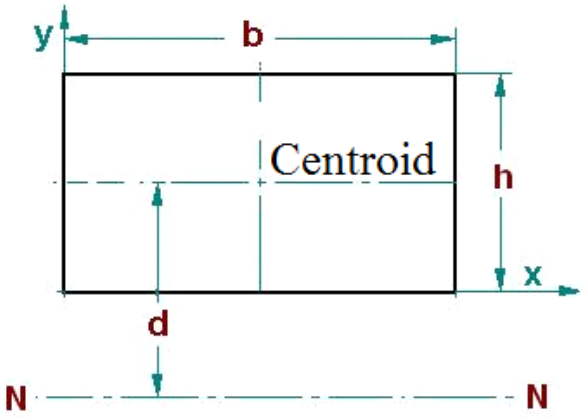
$I = I_c + Ad^2$

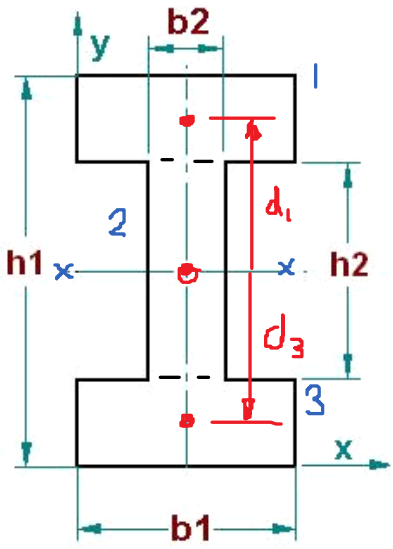
I = The second moment of area about non-centroidal plane
I_c = The second moment of area about its own centroid
A = Area of region
d = Distance from centroid to new plane

Q9: Find second moment of area Ixx about axis N-N where b = 18 , h = 4.9 and d = 6.2 mm.



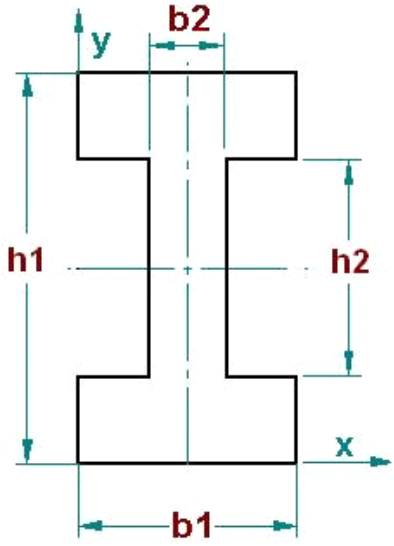
$I_c = bh^3/12$
$= 18*4.9\text{^}3/12 = 176.4735\ mm^4$
$I = I_c + Ad^2$
$= 176.4735 + (18*4.9)*(6.2\text{^}2)$
$= 3566.9\ mm4$

Q10: Find second moment of area Ixx about centroid, where b1 = 14 , h1 = 26 , b2 = 5.8 and h2 = 15 mm.

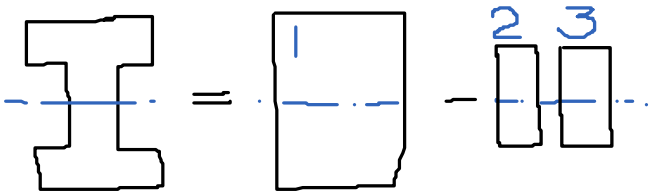


Split into 3 regions (elements)
E1 & E3: Top and bottom. E2 is middle web.
height = (h1-h2)/2 = (26-15)/2 = 5.5 mm
d1 = 15/2+5.5/2 = 10.25 mm
$I_1 = I_{c1} + Ad^2$
$I_{c1} = bh^3/12 = 14*5.5\text{^}3/12 = 194.1042\ mm^4$
$Ad^2 = (14*5.5)*(10.25\text{^}2) = 8089.8125\ mm^4$
$I_1 = 194.1042 + 8089.8125 = 8283.9167\ mm^4$
E3 is same as E1.
E2:
$I_1 = I_{c1} + \cancel{Ad^2}$ $d = 0$
$I_{c2} = bh^3/12 = 5.8*15\text{^}3/12 = 1631.25\ mm^4$
Total I = I1+I2+I3
= 8283.917 + 8283.917 +1631.25 = **18199.08 mm⁴**

Q10: Find second moment of area Ixx about centroid, where b1 = 14 , h1 = 26 , b2 = 5.8 and h2 = 15 mm.



Can we split this shape into elements that have the *same neutral axis*? Yes!



E1: $I=bh^3/12 = 14*26\text{^}3/12 = 20505.33\ mm^4$

E2: $I=bh^3/12$

b = (14-5.8)/2 = 4.1 mm

$I = 4.1*15\text{^}3/12 = 1153.125\ mm^4$

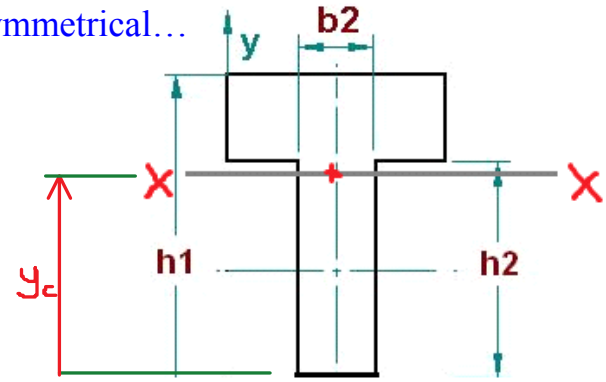
$Ixx = I_1 - I_2 - I_3$

$= 20505.33 - 1153.125 - 1153.125$

$= 18199.08\ mm^4$

This is a great shortcut **if** you can split up into elements that all have the same centroidal axis.

Not so easy if the compound shape is not symmetrical…
For example, this "T" beam.
Where is the Neutral Plane (x----x)?
Need to find the **centroid.**
Well, at least the y coordinate
of the centroid, y_c.



Finding Centroid

Wednesday, 27 March 2013
12:06 PM

$y_c = \Sigma(Ay)/\Sigma(A)$ $x_c = \Sigma(Ax)/\Sigma(A)$

y1 = 412/2 = 206 mm

y2 = (615-412)/2 + 412 = 513.5 mm

A1 = 335*412 = 138020 mm^2

A2 = 130*(615-412) = 26390 mm^2

A1y1 = 138020*206 = 28432120

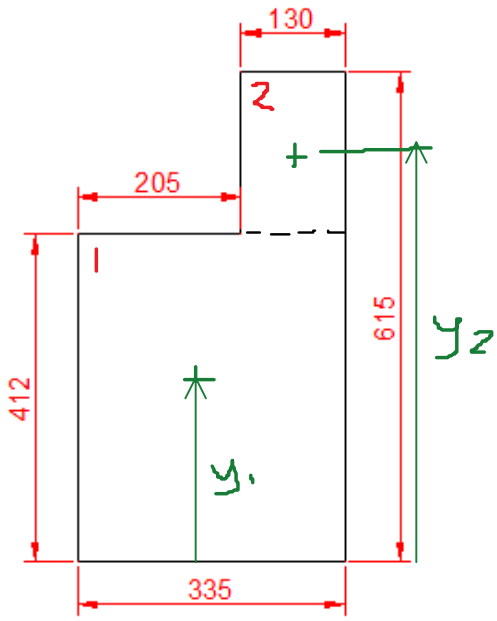
A2y2 = 26390*513.5 = 13551265

Σ(Ay) = 28432120+13551265 = 41983385

Σ(A) = 138020+26390 = 164410

$y_c = \Sigma(Ay)/\Sigma(A)$ = 41983385/164410

= 255.3579 mm



Element	A	y	Ay	x	Ax
	mm2	mm	mm3	mm	mm3
1	138020	206	28432120	167.5	23118350
2	26390	513.5	13551265	270	7125300
Total	164410		41983385		30243650
Centroid			**255.35786**		**183.9526**

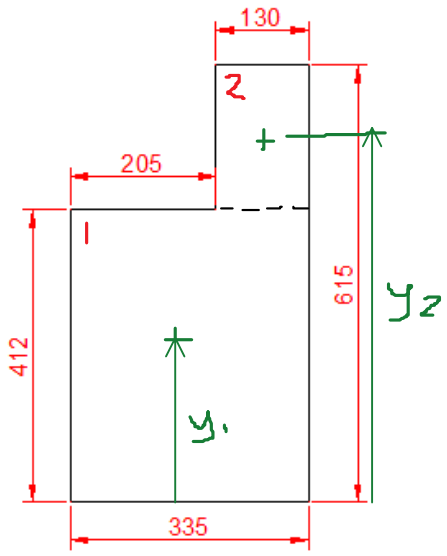
The centroid is where this piece of polycarbonate will balance.

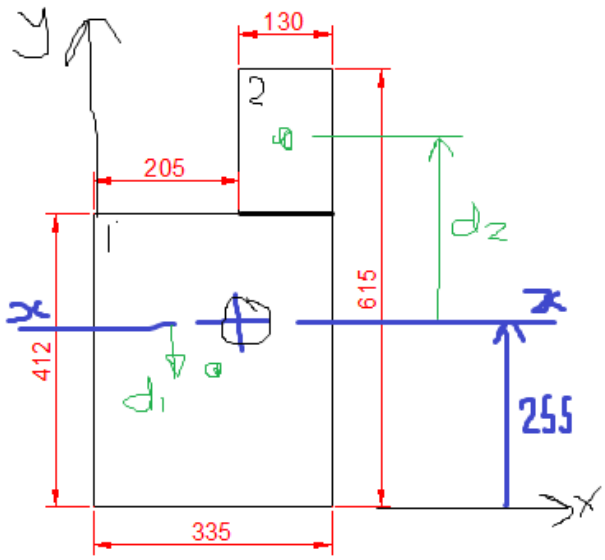
Finding Total Ixx

Wednesday, 27 March 2013
12:06 PM

$y_c = \Sigma(Ay)/\Sigma(A) = 41983385/164410$
$= 255.3579$ mm

The centroid is where this piece of polycarbonate will balance.





$I = I_c + Ad^2$

Element	A	y	Ay	Ic	d	Ad^2	I	
	mm2	mm	mm3	mm4	mm	mm4	mm4	
1	138020	206	28432120	1952338907	49.35786	336244032	2288582939	
2	26390	513.5	13551265	90625459.2	258.1421	1.759E+09	1849185570	
Total	164410		41983385	NO!		NO!	4137768509	
Centroid			255.35786					
							4137.76851	E6mm4

Finding Centroid for Composite Shape (p368)

Tuesday, 13 March 2012
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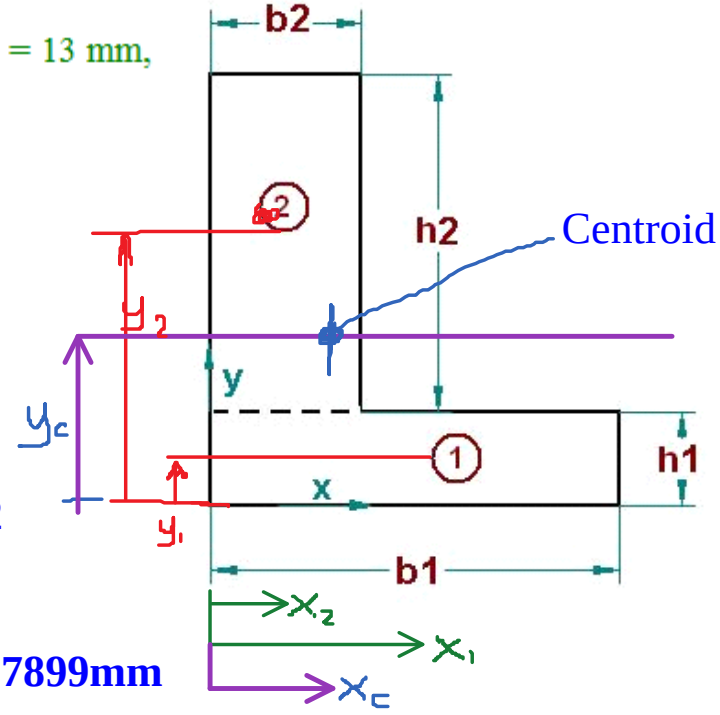
$I = I_c + Ad^2$

I = The second moment of area about non-centroidal plane
I_c = The second moment of area about its own centroid
A = Area of region
d = Distance from centroid to new plane

$y_c = \Sigma(Ay)/\Sigma(A)$

Q11: If b1 = 16 , h1 = 4.2 , b2 = 6.2 and h2 = 13 mm, find Yc. (Y coordinate of Centroid)



$y_c = \Sigma(Ay)/\Sigma(A)$
Element 1: $A_1 = 16*4.2 = 67.2\ mm^2$
$y_1 = 4.2/2 = 2.1\ mm$
$A_1y_1 = 67.2*2.1 = 141.12\ mm^3$
Element 2: $A_2 = 6.2*13 = 80.6\ mm^2$
$y_2 = 4.2 + 13/2 = 10.7\ mm$
$A_2y_2 = 80.6*10.7 = 862.42\ mm^3$
$\Sigma(Ay) = A_1y_1 + A_2y_2 = 141.12 + 862.42$
$= 1003.54\ mm^3$
$\Sigma(A) = 67.2+80.6 = 147.8\ mm^2$
$y_c = \Sigma(Ay)/\Sigma(A) = 1003.54/147.8 =$ **6.7899mm**

Table format

Element	A	y	Ay	x	Ax
	mm2	mm	mm3	mm	mm3
1	67.2	2.1	141.12	8	537.6
2	80.6	10.7	862.42	3.1	249.86
Total	147.8	n.a.	1003.54	n.a.	787.46
Centroid			**6.7899**		**5.328**

$y_c = \Sigma(Ay)/\Sigma(A)$ $x_c = \Sigma(Ax)/\Sigma(A)$

Finding Ixx for Composite Shape (p368)

Tuesday, 13 March 2012
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$I = I_c + Ad^2$

I = The second moment of area about non-centroidal plane
I_c = The second moment of area about its own centroid
A = Area of region
d = Distance from centroid to new plane

$y_c = \Sigma(Ay)/\Sigma(A)$ $x_c = \Sigma(Ax)/\Sigma(A)$

Q11: If b1 = 16 , h1 = 4.2 , b2 = 6.2 and h2 = 13 mm, find Yc. (Y coordinate of Centroid)

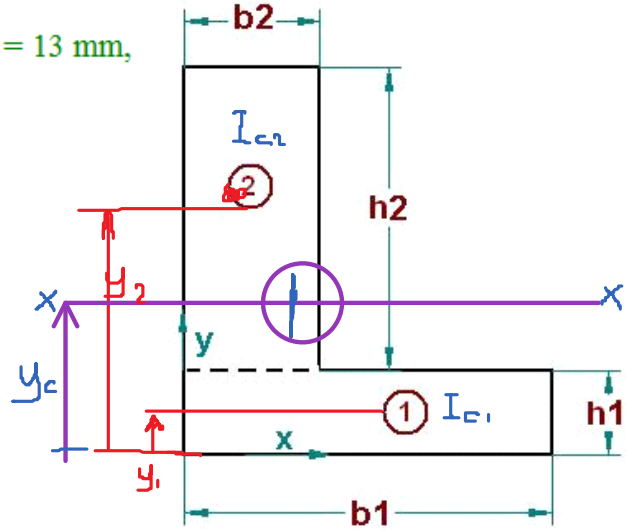
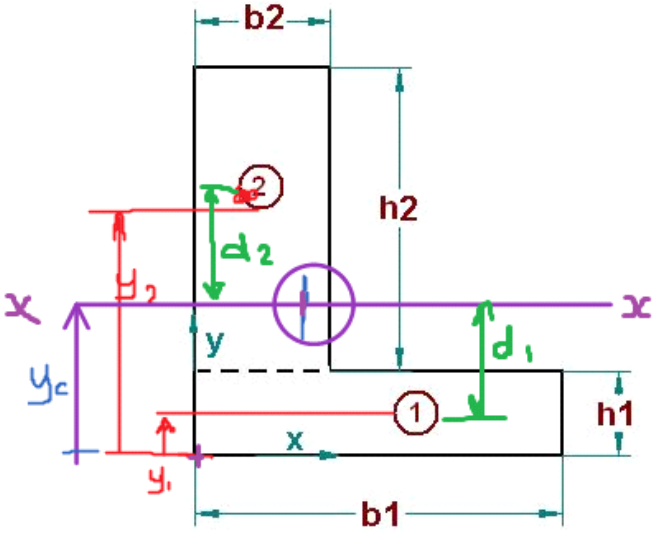


Table format *(Centroid)*

Element	A	y	Ay
	mm2	mm	mm3
1	67.2	2.1	141.12
2	80.6	10.7	862.42
Total	147.8	X	1003.54
Centroid			**6.7899**

Now find Ixx. $I = I_c + Ad^2$



Working

Ic_1 = 16*4.2^3/12 = 98.784 mm^4
Ic_2 = 6.2*13^3/12 = 1135.1167 mm^4
d_1 = yc - h1/2 = 6.7899-2.1 = 4.6899 mm
d_2 = 10.7 - 6.7899 = 3.9101 mm
$A_1d_1^2$ = 67.2 * 4.6899^2 = 1478.0749 mm^4
I_1= 98.784 + 1478.07 = 1576.854 mm^4
$A_2d_2^2$ = 80.6 *3.9101^2 = 1232.28 mm^4
I_2 = 1135.12 + 1232.28 = 2367.4 mm^4
Total I = $I_1 + I_2$ = 1576.854 + 2367.4 = 3944.254 mm^4

Element	A	y	Ay	Ic	d	Ad^2	I
	mm^2	mm	mm^3	mm^4	mm	mm^4	mm^4
1	67.2	2.1	141.12	98.784	4.6899	1478.07	1576.85
2	80.6	10.7	862.42	1135.12	3.9101	1232.28	2367.4
Total	147.8	n.a.	1003.54	n.a.	n.a.	n.a.	3944.254
Centroid		y_C =	**6.7899**				

I_{xx}

Centroid

$y_c = \Sigma(Ay)/\Sigma(A)$

Based on temporary axis

Ixx

$I = I_c + Ad^2$

Based on centroidal axis

Full Centroid Question

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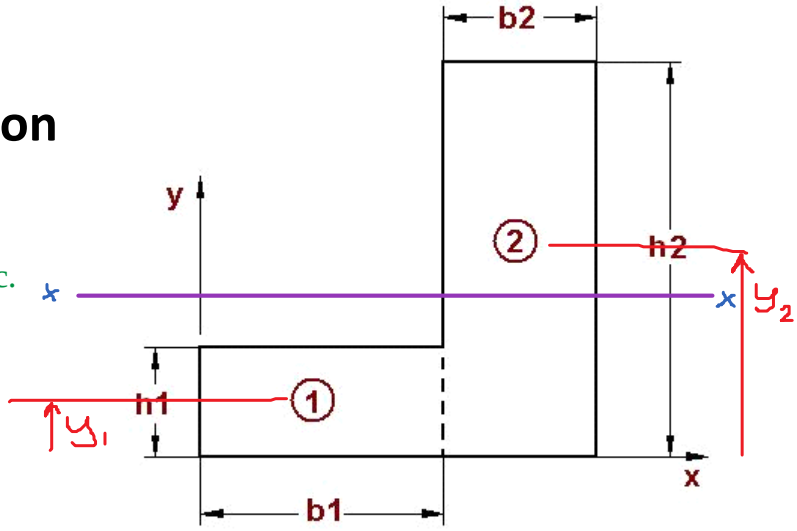
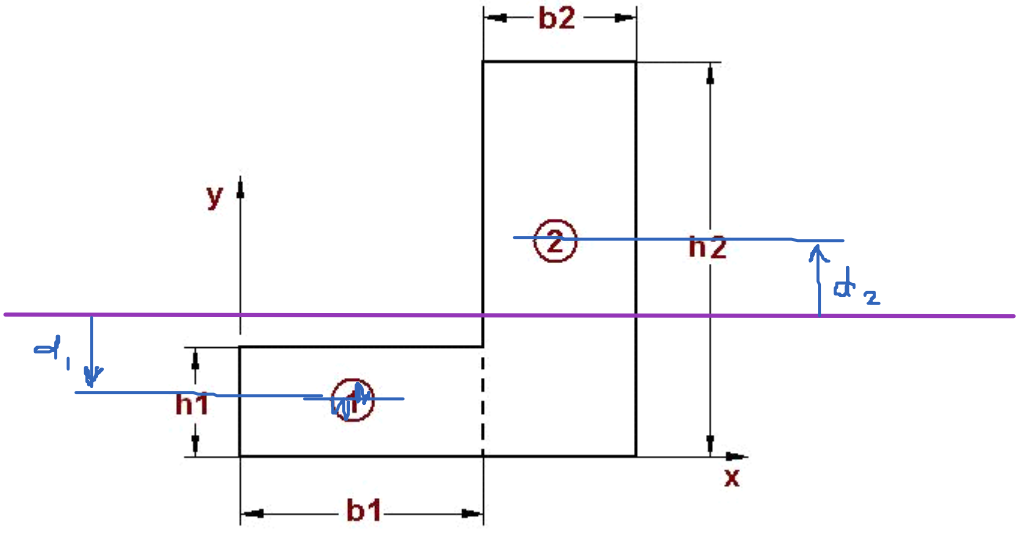
Q13: If b1 = 14 , h1 = 4.2 , b2 = 3.9 and h2 = 25 mm. (a) Find Yc. (Y coordinate of Centroid)



y1 = 4.2/2 = 2.1 mm
y2 = 25/2 = 12.5 mm
$y_c = \Sigma(Ay)/\Sigma(A)$
A1= 14*4.2 = 58.8 mm^2
A2 = 3.9*25 = 97.5 mm^2

Element	A	y	Ay
	mm^2	mm	mm^3
1	58.8	2.1	123.48
2	97.5	12.5	1218.75
Total	156.3	n.a.	1342.23
Centroid		yc =	**8.5875**

A1 + A2 = 58.8+97.5 = 156.3
A1*y1 = 58.8*2.1 = 123.48
A2*y2 = 97.5*12.5 = 1218.75
A1y1+A2y2 = 123.48+1218.75=1342.23
Yc = ΣAy/ΣA = 1342.23/156.3 = 8.5875



$\frac{bh^3}{12}$

$I = I_c + Ad^2$

Element	A	y	Ay	Ic	d	Ad^2	I
	mm^2	mm	mm^3	mm^4	mm	mm^4	mm^4
1	58.8	2.1	123.48	86.436	6.4875	2474.75	2561.19
2	97.5	12.5	1218.75	5078.125	3.9125	1492.50	6570.62
Total	156.3	n.a.	1342.23	n.a.	n.a.	n.a.	**9131.81**
Centroid		yc =	**8.5875**				

Ic1 = 14*4.2^3/12 = 86.436
Ic2 = 3.9*25^3/12 = 5078.125
d1 = 8.5875-2.1 = 6.4875
d2 = 12.5 - 8.5875 = 3.9125
$Ad1^2$ = 58.8*6.4875^2 = 2474.75
$Ad2^2$ = 97.5*3.9125^2 = 1492.4965
I1 = 86.436+2474.75 = 2561.19
I2 = 5078.125+1492.4965 = 6570.6215
Ixx = 2561.19+6570.6215 = 9131.8115